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: Transportation Model -1-5

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" 1941

" 1947 "

1951 1963 "

*Modify Distribution method (MODI)*

. 1954 *Stepping Stone*

*Assignment problem* 1955

1957

R.A.M. 1958 V.A.M.

. 1968

: The least cost transportation problem -2-1-5

:

.  $i$   $n$   $m$   $S_i$

.  $j$   $D_j$

.  $j$   $i$   $(i, j)$   $C_{ij}$

.  $j$   $i$   $X_{ij}$

.  $j$   $i$

:

$$\begin{aligned} \min . \quad & Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^n X_{ij} = a_i \\ & \sum_{i=1}^m X_{ij} = b_j \\ & X_{ij} \geq 0 \end{aligned}$$

:

*unbalanced*

.

-1

$$\cdot \sum_j b_j - \sum_i a_i$$

-2

$$\cdot \sum_i a_i - \sum_j b_j$$

:

*S.B.F.S.*

-1

-2

( )

-3

*S.B.F.S.* -2-1-5

:

*Northwest corner method*

-1

)  $X_{11}$

( )

$$X_{11} = \min.(a_1, b_1)$$

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*Least cost method*

-2

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**Vogel's Approximation Method ( V.A.M.)** -3

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**Russel's Approximation Method (R.A.M.)** -4

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(  $\bar{b}_j$  )

)

(  $\bar{a}_i$  )

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$\Delta_{ij} = C_{ij} - \bar{a}_i - \bar{b}_j$  :

-

$\Delta_{ij}$  ، ونعطي لمتغيرها أكبر كمية ممكنة والتي

-

تساوي  $\min.(a_i, b_j)$  .

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.1 -

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.2

: \_\_\_\_\_

: **Optimal Solution** **-3-1-5**

*S.B.F.S.*

*m n m+n-1*

: **Stepping Stone method** **-1**

$\bar{C}_{ij}$

$\bar{C}_{ij}$

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: **Multipliers method** **-2**

*Modified Distribution method (MODI)*

*Duality theory*

$V_j$

$j$

$U_i$

$i$

:  $X_{ij}$

$$U_i + V_j = C_{ij}$$

$(m+n)$  (  $(m+n-1)$  )  $(m+n-1)$

$(U_1=0)$

:  $X_{pq}$   $\bar{C}_{pq}$   
 $\bar{C}_{pq} = C_{pq} - (U_p + V_q)$

$\bar{C}_{pq}$

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**25 20 15**  $S_3, S_2, S_1$  : 1-

**15 12 10 8**  $C_4, C_3, C_2, C_1$

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: ( )

	$C_1$	$C_2$	$C_3$	$C_4$
$S_1$	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
$S_2$	<b>3</b>	<b>2</b>	<b>5</b>	<b>2</b>
$S_3$	<b>4</b>	<b>1</b>	<b>2</b>	<b>3</b>

$(25+20+15=60)$  :     

$C_5$   $(8+10+12+15=45)$

$(60-45=15)$

:      - S.B.F.S. -1

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	<b>2</b> 8	<b>3</b> 7	<b>4</b>	<b>5</b>	<b>0</b>	<b>15</b>
$S_2$	<b>3</b>	<b>2</b> 3	<b>5</b> 12	<b>2</b> 5	<b>0</b>	<b>20</b>
$S_3$	<b>4</b>	<b>1</b>	<b>2</b>	<b>3</b> 10	<b>0</b> 15	<b>25</b>
<b>Demand</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>

:

$$T.T.C. = 2*8 + 3*7 + 2*3 + 5*12 + 2*5 + 3*10 + 0*15 = 143$$

:

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	$\begin{matrix} 2 \\ 0 \end{matrix}$	$\begin{matrix} 3 \\ \end{matrix}$	$\begin{matrix} 4 \\ \end{matrix}$	$\begin{matrix} 5 \\ \end{matrix}$	$\begin{matrix} 0 \\ 15 \end{matrix}$	$15$
$S_2$	$\begin{matrix} 3 \\ 5 \end{matrix}$	$\begin{matrix} 2 \\ \end{matrix}$	$\begin{matrix} 5 \\ \end{matrix}$	$\begin{matrix} 2 \\ 15 \end{matrix}$	$\begin{matrix} 0 \\ \end{matrix}$	$20$
$S_3$	$\begin{matrix} 4 \\ 3 \end{matrix}$	$\begin{matrix} 1 \\ 10 \end{matrix}$	$\begin{matrix} 2 \\ 12 \end{matrix}$	$\begin{matrix} 3 \\ \end{matrix}$	$\begin{matrix} 0 \\ \end{matrix}$	$25$
<b>Demand</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>

:

$$T.T.C. = 2*0 + 0*15 + 3*5 + 2*15 + 4*3 + 1*10 + 2*12 = 91$$

: VAM

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply	P.C.
$S_1$	$\begin{matrix} 2 \\ 0 \end{matrix}$	$\begin{matrix} 3 \\ \end{matrix}$	$\begin{matrix} 4 \\ \end{matrix}$	$\begin{matrix} 5 \\ \end{matrix}$	$\begin{matrix} 0 \\ 15 \end{matrix}$	$15$	$\underline{2} \ 1 \ 1 \ \underline{3}$
$S_2$	$\begin{matrix} 3 \\ 5 \end{matrix}$	$\begin{matrix} 2 \\ \end{matrix}$	$\begin{matrix} 5 \\ \end{matrix}$	$\begin{matrix} 2 \\ 15 \end{matrix}$	$\begin{matrix} 0 \\ \end{matrix}$	$20$	$2 \ 0 \ 0 \ 1 \ 1$
$S_3$	$\begin{matrix} 4 \\ 3 \end{matrix}$	$\begin{matrix} 1 \\ 10 \end{matrix}$	$\begin{matrix} 2 \\ 12 \end{matrix}$	$\begin{matrix} 3 \\ \end{matrix}$	$\begin{matrix} 0 \\ \end{matrix}$	$25$	$1 \ 1 \ 2 \ 1 \ 1$
<b>Demand</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>	
<b>P.C.</b>	$\begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix}$	$\begin{matrix} 1 \\ 1 \\ 1 \\ \end{matrix}$	$\begin{matrix} 2 \\ \underline{2} \\ \underline{2} \\ \end{matrix}$	$\begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix}$	$\begin{matrix} 0 \\ \end{matrix}$		

:

$$T.T.C. = 2*0 + 0*15 + 3*5 + 2*15 + 4*3 + 1*10 + 2*12 = 91$$

: RAM

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	2 8	3	4	5	0 7	15
$S_2$	3	2	5	2 15	0 5	20
$S_3$	4	1 10	2 12	3	0 3	25
<b>Demand</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>

:

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$S_1$	-7	-5	-6	-5	-5
$S_2$	-6	-6	-5	-8	-5
$S_3$	-4	-6	-7	-6	-4

:  $C_4$

$X_{24}$

	$C_1$	$C_2$	$C_3$	$C_5$
$S_1$	-6	-4	-5	-4
$S_2$	-6	-6	-5	-5
$S_3$	-4	-6	-7	-4

:  $C_3$

$X_{33}$

	$C_1$	$C_2$	$C_5$
$S_1$	-5	-3	-3
$S_2$	-4	-4	-3
$S_3$	-4	-6	-4

:  $C_2$

$X_{32}$

	$C_1$	$C_5$
$S_1$	-4	-2
$S_2$	-4	-3
$S_3$	-4	-4

:  $S_3$

$X_{35}$

	$C_1$	$C_5$
$S_1$	-3	-2
$S_2$	-3	-3

$X_{15}$ ,

$S_2$

$X_{25}$

$X_{11}$

$$T.T.C. = 2*8 + 0*7 + 2*15 + 0*5 + 1*10 + 2*12 + 0*3 = 80$$

$$RAM < (91) VAM \leq (91) < (143) \quad (80)$$

. VAM RAM  
 ) VAM S.B.F.S.  
 RAM

$$No. of basic cells = m+n-1 = 5+3-1=7$$

: : Optimal solution -2  
 : Stepping stone -

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	2 0	3	4	5	0 15	15
$S_2$	3 5	2	5	2 15	0	20
$S_3$	4 3	1 10	2 12	3	0	25
<b>Demand</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>

$$\begin{aligned}
 X_{12} \rightarrow X_{32} \rightarrow X_{31} \rightarrow X_{11} & : \quad \bar{C}_{12} = 3 - 1 + 4 - 2 = 4 \\
 X_{13} \rightarrow X_{33} \rightarrow X_{31} \rightarrow X_{11} & : \quad \bar{C}_{13} = 4 - 2 + 4 - 2 = 4 \\
 X_{14} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} & : \quad \bar{C}_{14} = 5 - 2 + 3 - 2 = 4 \\
 X_{22} \rightarrow X_{32} \rightarrow X_{31} \rightarrow X_{21} & : \quad \bar{C}_{22} = 2 - 1 + 4 - 3 = 2 \\
 X_{23} \rightarrow X_{33} \rightarrow X_{31} \rightarrow X_{21} & : \quad \bar{C}_{23} = 5 - 2 + 4 - 3 = 4 \\
 X_{25} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21} & : \quad \bar{C}_{25} = 0 - 0 + 2 - 3 = -1 \\
 X_{34} \rightarrow X_{23} \rightarrow X_{21} \rightarrow X_{31} & : \quad \bar{C}_{34} = 3 - 2 + 3 - 4 = 0 \\
 X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{31} & : \quad \bar{C}_{35} = 0 - 0 + 2 - 4 = -2 \text{ most negative}
 \end{aligned}$$



entering variable

$\bar{C}_{35}$

leaving variable

$\cdot X_{35}$

$X_{ij}$

$X_{35}^+ \rightarrow X_{15}^- \rightarrow X_{11}^+ \rightarrow X_{31}^-$  :

:

$X_{31}$

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	2 3	3	4	5	0 12	15
$S_2$	3 5	2	5	2 15	0	20
$S_3$	4	1 10	2 12	3	0 3	25
<b>Demand</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>

$T.T.C. = 6 + 0 + 15 + 30 + 10 + 24 + 0 = 85$

$No. \text{ of basic cells} = 5 + 3 - 1 = 7$

$X_{12} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15}$  :  $\bar{C}_{12} = 3 - 1 + 0 - 0 = 2$   
 $X_{13} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15}$  :  $\bar{C}_{13} = 4 - 2 + 0 - 0 = 2$   
 $X_{14} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11}$  :  $\bar{C}_{14} = 5 - 2 + 3 - 2 = 4$   
 $X_{22} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21}$  :  $\bar{C}_{22} = 2 - 1 + 0 - 0 + 2 - 3 = 0$   
 $X_{23} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21}$  :  $\bar{C}_{23} = 5 - 2 + 0 - 0 + 2 - 3 = 2$   
 $X_{25} \rightarrow X_{21} \rightarrow X_{11} \rightarrow X_{15}$  :  $\bar{C}_{25} = 0 - 3 + 2 - 0 = -1$  negative  
 $X_{31} \rightarrow X_{11} \rightarrow X_{15} \rightarrow X_{35}$  :  $\bar{C}_{31} = 4 - 2 + 0 - 0 = 2$   
 $X_{34} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} \rightarrow X_{15} \rightarrow X_{35}$  :  $\bar{C}_{34} = 3 - 2 + 3 - 2 + 0 - 0 = 2$

:

$X_{21}$

$X_{25}$

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	2 8	3	4	5	0 7	15
$S_2$	3	2	5	2 15	0 5	20
$S_3$	4	1 10	2 12	3	0 3	25
<b>Demand</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>

$T.T.C. = 16 + 0 + 30 + 0 + 10 + 24 + 0 = 80$

$No. \text{ of basic cells} = 7$

$$\begin{aligned}
X_{12} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} & : \bar{C}_{12} = 3 - 1 + 0 - 0 = 2 \\
X_{13} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} & : \bar{C}_{13} = 4 - 2 + 0 - 0 = 2 \\
X_{14} \rightarrow X_{24} \rightarrow X_{25} \rightarrow X_{15} & : \bar{C}_{14} = 5 - 2 + 0 - 0 = 3 \\
X_{21} \rightarrow X_{25} \rightarrow X_{15} \rightarrow X_{11} & : \bar{C}_{21} = 3 - 0 + 0 - 2 = 1 \\
X_{22} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{25} & : \bar{C}_{22} = 2 - 1 + 0 - 0 = 1 \\
X_{23} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{25} & : \bar{C}_{23} = 5 - 2 + 0 - 0 = 3 \\
X_{31} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} & : \bar{C}_{31} = 4 - 0 + 0 - 2 = 2 \\
X_{34} \rightarrow X_{35} \rightarrow X_{25} \rightarrow X_{24} & : \bar{C}_{34} = 3 - 0 + 0 - 2 = 1
\end{aligned}$$

$$\begin{array}{ccc}
& & \bar{C}_{ij} \\
& & 8 \\
& & 15 \\
12 & 10 &
\end{array}$$

$V_j, U_i$

: Multipliers method

-2

$U_1=0$  :

$U_i + V_j = C_{ij}$  :

: VAM

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	2 0	3	4	5	0 15	15
$S_2$	3 5	2	5	2 15	0	20
$S_3$	4 3	1 10	2 12	3	0	25
Demand	8	10	12	15	15	60

T.T.C. = 91 and no. of basic cells = 7

$$\begin{aligned}
C_{11} = U_1 + V_1 = 2 & \Rightarrow V_1 = 2 & U_1=0 \\
C_{15} = U_1 + V_5 = 0 & \Rightarrow V_5 = 0 & U_1=0 \\
C_{21} = U_2 + V_1 = 3 & \Rightarrow U_2 = 1 & V_1=2 \\
C_{24} = U_2 + V_4 = 2 & \Rightarrow V_4 = 1 & U_2=1 \\
C_{31} = U_3 + V_1 = 4 & \Rightarrow U_3 = 2 & V_1=2 \\
C_{32} = U_3 + V_2 = 1 & \Rightarrow V_2 = -1 & U_3=2 \\
C_{33} = U_3 + V_3 = 2 & \Rightarrow V_3 = 0 & U_3=2
\end{aligned}$$

$$\begin{aligned}
 & \bar{C}_{ij} = C_{ij} - (U_i + V_j) \quad \bar{C}_{ij} \\
 \bar{C}_{12} &= C_{12} - (U_1 + V_2) = 3 - (0 + (-1)) = 4 \\
 \bar{C}_{13} &= C_{13} - (U_1 + V_3) = 4 - (0 + 0) = 4 \\
 \bar{C}_{14} &= C_{14} - (U_1 + V_4) = 5 - (0 + 1) = 4 \\
 \bar{C}_{22} &= C_{22} - (U_2 + V_2) = 2 - (0 - 1) = 2 \\
 \bar{C}_{23} &= C_{23} - (U_2 + V_3) = 5 - (1 + 0) = 4 \\
 \bar{C}_{25} &= C_{25} - (U_2 + V_5) = 0 - (1 + 0) = -1 \\
 \bar{C}_{34} &= C_{34} - (U_3 + V_4) = 3 - (2 + 1) = 0 \\
 \bar{C}_{35} &= C_{35} - (U_3 + V_5) = 0 - (2 + 0) = -2 \text{ most negative}
 \end{aligned}$$

$$\begin{aligned}
 & X_{35} \quad \bar{C}_{ij} \\
 & X_{35}^+ \rightarrow X_{15}^- \rightarrow X_{11}^+ \rightarrow X_{31}^- \\
 & X_{31}
 \end{aligned}$$

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$S_1$	2 3	3	4	5	0 12	15
$S_2$	3 5	2	5	2 15	0	20
$S_3$	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

$$T.T.C. = 6 + 0 + 15 + 30 + 10 + 24 + 0 = 85$$

$$\text{No. of basic cells} = m + n - 1 = 3 + 5 - 1 = 7$$

$$\bar{C}_{ij}$$

:

		$V_1=2$	$V_2=1$	$V_3=2$	$V_4=1$	$V_5=0$	Supply
		$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	
$U_1=0$	$S_1$	2 3	3 2	4 2	5 4	0 12	15
$U_2=1$	$S_2$	3 5	2 0	5 2	2 15	0 -1	20
$U_3=0$	$S_3$	4 2	1 10	2 12	3 2	0 3	25
Demand		8	10	12	15	15	60

$X_{21}$  $\bar{C}_{ij}$  $X_{25}$ 

:

		$V_1=2$	$V_2=1$	$V_3=2$	$V_4=2$	$V_5=0$	Supply
		$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	
$U_1=0$	$S_1$	2	3	4	5	0	15
		8	2	2	3	7	
$U_2=0$	$S_2$	3	2	5	2	0	20
		1	1	3	15	5	
$U_3=0$	$S_3$	4	1	2	3	0	25
		2	10	12	1	3	
<b>Demand</b>		<b>8</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>15</b>	<b>60</b>

$$T.T.C. = 16 + 0 + 30 + 0 + 10 + 24 + 0 = 80$$

)  $\bar{C}_{ij}$ 

:

(

8

15

12 10