
: Transportation Model -1-5

)

() (

()

" 1941

" 1947 "

1951 1963 "

Modify Distribution method (MODI)

. 1954 *Stepping Stone*

Assignment problem 1955

1957

R.A.M. 1958 V.A.M.

. 1968

: The least cost transportation problem -2-1-5

	:	n	m	
	i			S_i
	j			D_j
j	i	(i, j)		C_{ij}
	j	i		X_{ij}
	j	i		
	:			

()

: Vogel's Approximation Method (V.A.M.) -3

:

-

-

()

()

.1 -

()

.2

()

.3

()

)

()

()

()

: Russel's Approximation Method (R.A.M.) -4

)

:

(

(\bar{b}_j)

)

(\bar{a}_i)

)

-

. $\Delta_{ij} = C_{ij} - \bar{a}_i - \bar{b}_j$:

-

Δ_{ij} ، ونعطي لمتغيرها أكبر كمية ممكنة والتي

-

. تساوي $\min.(a_i, b_j)$

()

-

()

()

.1 -

()

()

.2

: _____

: Optimal Solution -3-1-5

S.B.F.S.

m n $m+n-1$

:

: Stepping Stone method -1

\bar{C}_{ij}

\bar{C}_{ij}

()

)

(

()

: Multipliers method -2

Modified Distribution method (MODI)

. Duality theory

V_j

j

U_i

i

: X_{ij}

$$U_i + V_j = C_{ij}$$

$(m+n)$ ($(m+n-1)$) $(m+n-1)$

$(U_1=0)$

: X_{pq} \bar{C}_{pq}
 $\bar{C}_{pq} = C_{pq} - (U_p + V_q)$

\bar{C}_{pq}

()

)

.

25 20 15 S_3, S_2, S_1 : 1-

15 12 10 8 C_4, C_3, C_2, C_1

()

:

()

	C_1	C_2	C_3	C_4
S_1	2	3	4	5
S_2	3	2	5	2
S_3	4	1	2	3

$(25+20+15=60)$

: ___

C_5

$(8+10+12+15=45)$

$(60-45=15)$

:

- S.B.F.S.

-1

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 8	3 7	4	5	0	15
S_2	3	2 3	5 12	2 5	0	20
S_3	4	1	2	3 10	0 15	25
Demand	8	10	12	15	15	60

:

$$T.T.C. = 2*8 + 3*7 + 2*3 + 5*12 + 2*5 + 3*10 + 0*15 = 143$$

:

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	$\begin{array}{ c } \hline 2 \\ \hline 0 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 4 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline 15 \end{array}$	15
S_2	$\begin{array}{ c } \hline 3 \\ \hline 5 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 15 \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	20
S_3	$\begin{array}{ c } \hline 4 \\ \hline 3 \end{array}$	$\begin{array}{ c } \hline 1 \\ \hline 10 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 12 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	25
Demand	8	10	12	15	15	60

:

$$T.T.C. = 2*0 + 0*15 + 3*5 + 2*15 + 4*3 + 1*10 + 2*12 = 91$$

: VAM

	C_1	C_2	C_3	C_4	C_5	Supply	P.C.
S_1	$\begin{array}{ c } \hline 2 \\ \hline 0 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 4 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline 15 \end{array}$	15	$\underline{2} \ 1 \ 1 \ \underline{3}$
S_2	$\begin{array}{ c } \hline 3 \\ \hline 5 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline \end{array}$	$\begin{array}{ c } \hline 5 \\ \hline \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 15 \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	20	$2 \ 0 \ 0 \ 1 \ 1$
S_3	$\begin{array}{ c } \hline 4 \\ \hline 3 \end{array}$	$\begin{array}{ c } \hline 1 \\ \hline 10 \end{array}$	$\begin{array}{ c } \hline 2 \\ \hline 12 \end{array}$	$\begin{array}{ c } \hline 3 \\ \hline \end{array}$	$\begin{array}{ c } \hline 0 \\ \hline \end{array}$	25	$1 \ 1 \ 2 \ 1 \ 1$
Demand	8	10	12	15	15	60	
P.C.	$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$	$\begin{array}{c} 1 \\ 1 \\ 1 \end{array}$	$\begin{array}{c} 2 \\ \underline{2} \\ \underline{2} \end{array}$	$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$	$\begin{array}{c} 0 \\ \end{array}$		

:

$$T.T.C. = 2*0 + 0*15 + 3*5 + 2*15 + 4*3 + 1*10 + 2*12 = 91$$

: RAM

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 8	3	4	5	0 7	15
S_2	3	2	5	2 15	0 5	20
S_3	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

:

	C_1	C_2	C_3	C_4	C_5
S_1	-7	-5	-6	-5	-5
S_2	-6	-6	-5	-8	-5
S_3	-4	-6	-7	-6	-4

: C_4

X_{24}

	C_1	C_2	C_3	C_5
S_1	-6	-4	-5	-4
S_2	-6	-6	-5	-5
S_3	-4	-6	-7	-4

: C_3

X_{33}

	C_1	C_2	C_5
S_1	-5	-3	-3
S_2	-4	-4	-3
S_3	-4	-6	-4

: C_2

X_{32}

	C_1	C_5
S_1	-4	-2
S_2	-4	-3
S_3	-4	-4

: S_3

X_{35}

	C_1	C_5
S_1	-3	-2
S_2	-3	-3

X_{15} ,

S_2

X_{25}

X_{11}

$$T.T.C. = 2*8 + 0*7 + 2*15 + 0*5 + 1*10 + 2*12 + 0*3 = 80$$

$$RAM < (91) VAM \leq (91) < (143) \quad (80)$$

. VAM RAM
) VAM S.B.F.S.
 RAM

$$No. of basic cells = m+n-1 = 5+3-1=7$$

: : Optimal solution -2
 : Stepping stone -

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 0	3	4	5	0 15	15
S_2	3 5	2	5	2 15	0	20
S_3	4 3	1 10	2 12	3	0	25
Demand	8	10	12	15	15	60

$$\begin{aligned}
 X_{12} \rightarrow X_{32} \rightarrow X_{31} \rightarrow X_{11} & : \quad \bar{C}_{12} = 3 - 1 + 4 - 2 = 4 \\
 X_{13} \rightarrow X_{33} \rightarrow X_{31} \rightarrow X_{11} & : \quad \bar{C}_{13} = 4 - 2 + 4 - 2 = 4 \\
 X_{14} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} & : \quad \bar{C}_{14} = 5 - 2 + 3 - 2 = 4 \\
 X_{22} \rightarrow X_{32} \rightarrow X_{31} \rightarrow X_{21} & : \quad \bar{C}_{22} = 2 - 1 + 4 - 3 = 2 \\
 X_{23} \rightarrow X_{33} \rightarrow X_{31} \rightarrow X_{21} & : \quad \bar{C}_{23} = 5 - 2 + 4 - 3 = 4 \\
 X_{25} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21} & : \quad \bar{C}_{25} = 0 - 0 + 2 - 3 = -1 \\
 X_{34} \rightarrow X_{23} \rightarrow X_{21} \rightarrow X_{31} & : \quad \bar{C}_{34} = 3 - 2 + 3 - 4 = 0 \\
 X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{31} & : \quad \bar{C}_{35} = 0 - 0 + 2 - 4 = -2 \text{ most negative}
 \end{aligned}$$

entering variable

\bar{C}_{35}

leaving variable

$\cdot X_{35}$

$X_{ij} \quad X_{35}^+ \rightarrow X_{15}^- \rightarrow X_{11}^+ \rightarrow X_{31}^- :$

:

X_{31}

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 3	3	4	5	0 12	15
S_2	3 5	2	5	2 15	0	20
S_3	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

$T.T.C. = 6 + 0 + 15 + 30 + 10 + 24 + 0 = 85$

$No. \text{ of basic cells} = 5 + 3 - 1 = 7$

$X_{12} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} : \bar{C}_{12} = 3 - 1 + 0 - 0 = 2$
 $X_{13} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} : \bar{C}_{13} = 4 - 2 + 0 - 0 = 2$
 $X_{14} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} : \bar{C}_{14} = 5 - 2 + 3 - 2 = 4$
 $X_{22} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21} : \bar{C}_{22} = 2 - 1 + 0 - 0 + 2 - 3 = 0$
 $X_{23} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} \rightarrow X_{21} : \bar{C}_{23} = 5 - 2 + 0 - 0 + 2 - 3 = 2$
 $X_{25} \rightarrow X_{21} \rightarrow X_{11} \rightarrow X_{15} : \bar{C}_{25} = 0 - 3 + 2 - 0 = -1 \text{ negative}$
 $X_{31} \rightarrow X_{11} \rightarrow X_{15} \rightarrow X_{35} : \bar{C}_{31} = 4 - 2 + 0 - 0 = 2$
 $X_{34} \rightarrow X_{24} \rightarrow X_{21} \rightarrow X_{11} \rightarrow X_{15} \rightarrow X_{35} : \bar{C}_{34} = 3 - 2 + 3 - 2 + 0 - 0 = 2$

:

X_{21}

X_{25}

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 8	3	4	5	0 7	15
S_2	3	2	5	2 15	0 5	20
S_3	4	1 10	2 12	3	0 3	25
Demand	8	10	12	15	15	60

$T.T.C. = 16 + 0 + 30 + 0 + 10 + 24 + 0 = 80$

$No. \text{ of basic cells} = 7$

$$\begin{aligned}
X_{12} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{15} & : \bar{C}_{12} = 3 - 1 + 0 - 0 = 2 \\
X_{13} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{15} & : \bar{C}_{13} = 4 - 2 + 0 - 0 = 2 \\
X_{14} \rightarrow X_{24} \rightarrow X_{25} \rightarrow X_{15} & : \bar{C}_{14} = 5 - 2 + 0 - 0 = 3 \\
X_{21} \rightarrow X_{25} \rightarrow X_{15} \rightarrow X_{11} & : \bar{C}_{21} = 3 - 0 + 0 - 2 = 1 \\
X_{22} \rightarrow X_{32} \rightarrow X_{35} \rightarrow X_{25} & : \bar{C}_{22} = 2 - 1 + 0 - 0 = 1 \\
X_{23} \rightarrow X_{33} \rightarrow X_{35} \rightarrow X_{25} & : \bar{C}_{23} = 5 - 2 + 0 - 0 = 3 \\
X_{31} \rightarrow X_{35} \rightarrow X_{15} \rightarrow X_{11} & : \bar{C}_{31} = 4 - 0 + 0 - 2 = 2 \\
X_{34} \rightarrow X_{35} \rightarrow X_{25} \rightarrow X_{24} & : \bar{C}_{34} = 3 - 0 + 0 - 2 = 1
\end{aligned}$$

$$\begin{array}{ccc}
& & \bar{C}_{ij} \\
& & 8 \\
& & 15 \\
12 & 10 &
\end{array}$$

V_j, U_i

: Multipliers method

-2

$$U_i = 0 :$$

$$U_i + V_j = C_{ij} :$$

: VAM

	C_1	C_2	C_3	C_4	C_5	Supply
S_1	2 0	3	4	5	0 15	15
S_2	3 5	2	5	2 15	0	20
S_3	4 3	1 10	2 12	3	0	25
Demand	8	10	12	15	15	60

T.T.C. = 91 and no. of basic cells = 7

$$\begin{aligned}
C_{11} = U_1 + V_1 = 2 & \Rightarrow V_1 = 2 & U_1=0 \\
C_{15} = U_1 + V_5 = 0 & \Rightarrow V_5 = 0 & U_1=0 \\
C_{21} = U_2 + V_1 = 3 & \Rightarrow U_2 = 1 & V_1=2 \\
C_{24} = U_2 + V_4 = 2 & \Rightarrow V_4 = 1 & U_2=1 \\
C_{31} = U_3 + V_1 = 4 & \Rightarrow U_3 = 2 & V_1=2 \\
C_{32} = U_3 + V_2 = 1 & \Rightarrow V_2 = -1 & U_3=2 \\
C_{33} = U_3 + V_3 = 2 & \Rightarrow V_3 = 0 & U_3=2
\end{aligned}$$

X_{21} \bar{C}_{ij} X_{25}

:

		$V_1=2$	$V_2=1$	$V_3=2$	$V_4=2$	$V_5=0$	Supply
		C_1	C_2	C_3	C_4	C_5	
$U_1=0$	S_1	2	3	4	5	0	15
		8	2	2	3	7	
$U_2=0$	S_2	3	2	5	2	0	20
		1	1	3	15	5	
$U_3=0$	S_3	4	1	2	3	0	25
		2	10	12	1	3	
Demand		8	10	12	15	15	60

$$T.T.C. = 16 + 0 + 30 + 0 + 10 + 24 + 0 = 80$$

) \bar{C}_{ij}

:

(

8

15

12 10