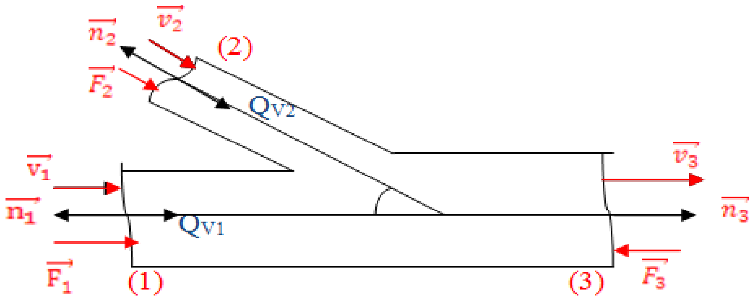
Université ZIANE Achour de Djelfa **2019/2020**
Faculté des sciences et de la technologie
Mécaniques des fluides

Solution de TD MDF

EXERCICE 01 :



Le théorème de quantité de mouvement:

$$\underbrace{\iint \rho \vec{v_i}\,(\vec{v_1}\,.\,\overrightarrow{n_{ext}})d\delta}_{I} = \underbrace{\sum \overrightarrow{F_{ext}}}_{II} \quad (*)$$

Le terme I :

$$I = \iint \rho\vec{v_1}\underbrace{(\vec{v_1}\,.\,\vec{n_1})}_{-v_1}\,d\delta + \iint \rho\vec{v_2}\underbrace{(\vec{v_2}\,.\,\vec{n_2})}_{-v_2}\,d\delta + \iint \rho\vec{v_3}\underbrace{(\vec{v_3}\,.\,\vec{n_3})}_{v_3}\,d\delta$$

$$I = -\rho v_1\vec{v_1}\iint d\delta - \rho v_2\vec{v_2}\iint d\delta + \rho v_3\vec{v_3}\iint d\delta$$

$$\boxed{I = -\rho v_1\vec{v_1}\,S_1 - \rho v_2\vec{v_2}\,S_2 + \rho v_3\vec{v_3}\,S_3}$$

Le terme II :

$$\sum \overrightarrow{F_{ext}} = M.\vec{g} + \vec{F_1} + \vec{F_2} + \vec{F_3} - \vec{R}$$

$$(*) \leftrightarrow -\rho v_1\vec{v_1}\,S_1 - \rho v_2\vec{v_2}\,S_2 + \rho v_3\vec{v_3}\,S_3 = M.\vec{g} + \vec{F_1} + \vec{F_2} + \vec{F_3} - \vec{R}$$

La projection selon l'axe $\vec{ox}$:

$$-\rho {v_1}^2\,S_1 - \rho {v_2}^2\,S_2\cos 30 + \rho {v_3}^2\,S_3 = 0 + F_1 + F_2\cos 30 - F_3 - R_x$$

$$R_x = \rho {v_1}^2\,S_1 + \rho {v_2}^2\,S_2\cos 30 - \rho {v_3}^2\,S_3 + F_1 + F_2\cos 30 - F_3$$

$$R_x = \rho {v_1}^2\,S_1 + \rho {v_2}^2\,S_2\cos 30 - \rho {v_3}^2\,S_3 + P_1S_1 + P_2S_2\cos 30 - P_3S_3$$

Calcule des vitesses :

$$q_{v_3} = q_{v_1} + q_{v_2} = 0.282\,m^3/s$$

$$v_1 = \frac{4q_{v_1}}{\pi D_1^2} = \frac{4 \times 0.201}{\pi (0.4)^2} = 1.6\ m/s$$

$$v_2 = \frac{4q_{v_2}}{\pi D_2^2} = \frac{4 \times 0.081}{\pi (0.25)^2} = 1.65\ m/s$$

$$v_1 = \frac{4q_{v_1}}{\pi D_1^2} = \frac{4 \times 0.282}{\pi (0.4)^2} = 2.24\ m/s$$

$$\begin{aligned} \mathrm{R_x} = {} & 6.376 \times 10^5 \times 0.125 + 6.376 \times 10^5 \times 0.049 \times \cos 30 - 6.278 \times 10^5 \times 0.125 \\ & + 10^3 \times 0.125 \times (1.6)^2 + 10^3 \times (1.65)^2 \times 0.049 \times \cos 30 \\ & - 10^3 \times (2.24)^2 \times 0.125 \end{aligned}$$

$$\mathrm{R_x} = \cdots N$$

La projection selon l'axe $\overrightarrow{oz}$:

$$\rho {v_2}^2\ \mathrm{S_2} \sin 30 = -\mathrm{Mg} - \mathrm{F_2} \sin 30 - \mathrm{R_z}$$

$$\mathrm{R_z} = -\mathrm{Mg} - \mathrm{F_2} \sin 30 - \rho {v_2}^2\ \mathrm{S_2} \sin 30$$

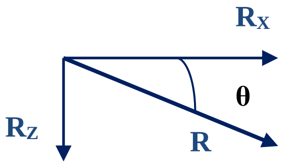
En négligent le poids devant les autres forces :

$$\mathrm{R_z} = -\mathrm{P_2 S_2} \sin 30 - \rho {v_2}^2\ \mathrm{S_2} \sin 30$$

$$\mathrm{R_z} = -6.376 \times 10^5 \times 0.049\ \sin 30 - 10^3 \times (1.65)^2 \times 0.049 \times \sin 30$$

$$\mathrm{R_z} = \cdots N$$

$$\mathrm{R} = \sqrt{R_X^2 + R_Z^2} = \cdots N$$

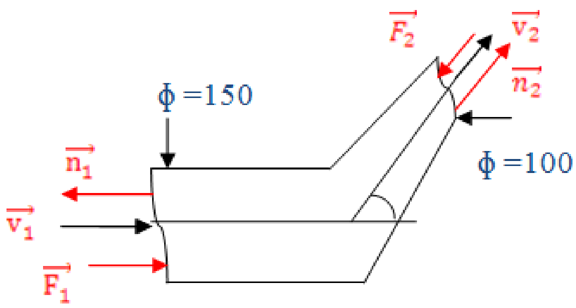


Et : $\theta = \cdots$

<u>EXERCICE 02 :</u>

Donnée :

$q_v = 40\frac{l}{s} = 0.040\ m^3/s$
$\mathrm{P_{1ef}} = 4\ \mathrm{bars} = 4 \times 10^5 \mathrm{Pa}$
$\mathrm{P_{2ef}} = 4 \times 10^4 \mathrm{Pa}$
$\mathrm{D_1} = 150\mathrm{mm} = 0.15\mathrm{m}$
$\mathrm{D_1} = 100\mathrm{mm} = 0.1\mathrm{m}$
$\alpha = 60°$



Le théorème de quantité de mouvement:

$$\underbrace{\iint \rho \vec{v_i}\, (\vec{v_i} \,.\, \overrightarrow{n_{ext}}) d\delta}_{I} = \underbrace{\sum \overrightarrow{F_{ext}}}_{II} \quad (*)$$

<u>Le terme I :</u>

$$I = \iint \rho\vec{v_1}\underbrace{(\vec{v_1}.\vec{n_1})}_{-v_1}\,d\delta + \iint \rho\vec{v_2}\underbrace{(\vec{v_2}.\vec{n_2})}_{+v_2}\,d\delta$$

$$I = -\rho v_1\vec{v_1}\iint d\delta + \rho v_2\vec{v_2}\iint d\delta$$

$$I = -\rho v_1\vec{v_1}\,S_1 - \rho v_2\vec{v_2}\,S_2 = q_m(\vec{v_2} - \vec{v_1})$$

Avec : $q_m = \rho v_1\,S_1 = \rho v_2\,S_2 = cst$

Le terme II :

$$\sum \overrightarrow{F_{ext}} = M.\vec{g} + \vec{F_1} + \vec{F_2} - \vec{R}$$

$$q_m(\vec{v_2} - \vec{v_1}) = M.\vec{g} + \vec{F_1} + \vec{F_2} - \vec{R}$$

La projection selon l'axe $\overrightarrow{ox}$:

$$q_m(v_2\cos 60 - v_1) = F_1 - F_2\cos 60 - R_X$$

$$R_X = F_1 - F_2\cos 60 + q_m(v_1 - v_2\cos 60)$$

$$R_X = P_1S_1 - P_2S_2\cos 60 + q_m(v_1 - v_2\cos 60)$$

$$S_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.15)^2}{4} = 0.0176m^2$$

$$S_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.1)^2}{4} = 0.0078m^2$$

$q_v = v_1\,S_1 = v_2\,S_2$

$$\Longrightarrow \begin{cases} v_1 = \frac{q_v}{S_1} = \frac{0.04}{0.0176} = 2.263\ m/s \\ v_2 = \frac{q_v}{S_2} = \frac{0.04}{0.0078} = 5.092\frac{m}{s} \end{cases}$$

$R_X = 4\times10^5\times0.0176 - 4\times10^4\times0.0078\cos 60 + 0.04\times10^3\times(2.263 - 5.092\cos 60)$

$$R_X = \cdots N$$

La projection seln l'axe $\overrightarrow{oz}$:

$$q_m(v_2\sin 60) = -F_2\sin 60 - R_z$$

$$R_z = -F_2\sin 60 - q_m(v_2\sin 60)$$

$$R_z = -P_2S_2\sin 60 - \rho q_v(v_2\sin 60)$$

$$R_z = -4\times10^4\times0.0078\times\sin 60 - 10^3\times0.04\times5.092\times\sin 60$$

$$R_z = \cdots N$$

$$R = \sqrt{R_X^2 + R_Z^2} = \cdots N$$

Et : $\theta = \cdots$