Exercice 1

1) $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{29,8 \times 365,25 \times 24 \times 3600}$

$= 7,63 \times 10^{-10}\ s^{-1}$

2) $A_0 = \lambda N_0 = \lambda \frac{m_0}{M} N_A$

$= 5,1 \times 10^5\ Bq = 0,14\ Ci$

3) $N(t) = N_0 e^{-\lambda t}$

$\frac{m(t)}{M} N_A = \frac{m_0}{M} N_A e^{-\lambda t}$

d'où : $m(t) = m_0 e^{-\lambda t}$

$\frac{m}{m_0} = \frac{0,25}{1} = \frac{1}{4} = e^{-\lambda t}$

d'où $-\ln 4 = -\lambda t$

et $\lambda t = 2 \ln 2 \Rightarrow t = \frac{2 \ln 2}{\lambda}$

$t = \frac{2 \ln 2}{\ln 2} \times t_{1/2} = 2 t_{1/2} = 57,6\ a$

$A = A_0 e^{-\lambda t} = A_0 e^{-\frac{\ln 2}{t_{1/2}} \times 2 t_{1/2}}$

$= A_0 e^{-\ln 4} = \frac{A_0}{4} = 1,27 \times 10^{9}\ Bq$

$= 0,034\ Ci$

Exercice 2

$A = \lambda N = \frac{\ln 2}{t_{1/2}} \times \frac{m}{M} \times N_A$

d'où :

$t_{1/2} = \frac{\ln 2}{A} \times \frac{m}{M} \times N_A$

$= \frac{\ln 2}{89} \times \frac{0,151}{15936} \times 6,02 \times 10^{23}$

$= 4,71 \times 10^{17}\ s = 1,4925 \times 10^{11}\ a$

Exercice 3

$A = \frac{15,3}{60} \times 70 \times 10^3 \times 0,19$

$= 3213\ Bq$

Exercice 5

a) $^{232}_{94}Pu \rightarrow {}^{232}_{92}U + {}^{4}_{2}\alpha$

$\mathcal{Q} = (m_{Pu} - m_U - m_\alpha) c^2$

$= 5,87\ MeV > 0$

$\Rightarrow$ désintégration possible.

b) Nous avons : $p_U = p_\alpha$

d'où : $p_U^2 = p_\alpha^2$

Donc : $2 m_U T_U = 2 m_\alpha T_\alpha$

et $T_U = \frac{m_\alpha}{m_U} T_\alpha$

$\mathcal{Q} = T_U + T_\alpha = \left(\frac{m_\alpha}{m_U} + 1\right) T_\alpha$

$= \left(\frac{m_\alpha + m_U}{m_U}\right) T_\alpha$

d'où : $T_\alpha = \frac{m_U}{m_\alpha + m_U} \mathcal{Q}$

$= 5,77\ MeV$ (98%)

c) $T_U = \mathcal{Q} - T_\alpha = 0,01\ MeV$ (2%)

Exercice 6

$^{6}_{2}He \rightarrow {}^{6}_{3}Li + {}^{0}_{-1}e^- + \bar{\nu}_e$

$\mathcal{Q} = (m_{He} - m_{Li} - m_e - m_{\bar{\nu}_e})$

$\simeq (m_{He} - m_{Li} - m_e) c^2$

$= T_{Li} + T_{e^-} + T_{\bar{\nu}_e} = T_{e^-}^{max}$

$m_{He} = \frac{T_e^{max}}{c^2} + m_{Li} + m_e$

$= \frac{1,983 + 0,511}{931,5} + 6,015121$

$= 6,017798\ u$

Exercice 7

$^{29}_{13}Al \rightarrow {}^{29}_{14}Si + {}^{0}_{-1}e^- + \bar{\nu}_e$

$^{237}_{93}Np \rightarrow {}^{233}_{91}Pa + {}^{4}_{2}\alpha$

$^{20}_{9}F^* \rightarrow {}^{20}_{9}F + {}^{0}_{0}\gamma$

$^{34}_{19}Ar \rightarrow {}^{34}_{17}Cl + {}^{0}_{+1}e^+ + \nu_e$

$^{119}_{51}Sb + {}^{0}_{-1}e^- \rightarrow {}^{119}_{50}Sn + \nu_e$

Exercice 8

$Ra \rightarrow Ac \rightarrow (3)$

a) $\frac{dx}{dt} = -\lambda x$

d'où : $\frac{dx}{x} = -\lambda dt$

et $\ln x = -\lambda t + ct$

$x(t) = x_0 e^{-\lambda t}$

$$\frac{dy}{dt} = -\lambda' y + \lambda x$$

$$\frac{dy}{dt} + \lambda' y = \lambda x_0 e^{-\lambda t} \quad \ldots\ldots \quad (1)$$

$$* \quad \frac{dy}{dt} + \lambda' y = 0$$

$$\frac{dy}{dt} = -\lambda' y \quad \text{et} \quad y(t) = K_1 e^{-\lambda' t}$$

$$* \quad y(t) = K_1(t)\, e^{-\lambda' t}$$

$$\frac{dy}{dt} = K_1' e^{-\lambda' t} - \lambda' K_1 e^{-\lambda' t} \rightarrow (1)$$

$$K_1' e^{-\lambda' t} - \lambda' K_1 e^{-\lambda' t} + \lambda' K_1 e^{-\lambda' t} = \lambda x_0 e^{-\lambda t}$$

$$K_1' e^{-\lambda' t} = \lambda x_0 e^{-\lambda t} \quad \text{d'où :}$$

$$K_1' = \lambda x_0 e^{(\lambda' - \lambda) t} \quad \text{d'où :}$$

$$K_1 = \frac{\lambda x_0}{\lambda' - \lambda} e^{(\lambda' - \lambda) t} + K_2$$

Donc :

$$y(t) = \frac{\lambda x_0}{\lambda' - \lambda} e^{-\lambda t} + K_2 e^{-\lambda' t}$$

$t = 0$, $y = 0$ d'où

$$K_2 = -\frac{\lambda x_0}{\lambda' - \lambda}$$

Enfin :

$$y(t) = \frac{\lambda x_0}{\lambda' - \lambda} \left(e^{-\lambda t} - e^{-\lambda' t} \right)$$

b) $\frac{m_{Ac}}{M_{Ac}} N_A = \frac{m_0}{M_{Ra}} N_A \frac{\lambda}{\lambda' - \lambda} \left(e^{-\lambda t} - e^{-\lambda' t} \right)$

$M_{Ac} \simeq M_{Ra}$

$$m_{Ac} = m_0 \frac{\lambda}{\lambda' - \lambda} \left(e^{-\lambda t} - e^{-\lambda' t} \right)$$

$$= m_0 \left(\frac{T'}{T - T'} \right) \left(e^{-\ln 2 \cdot \frac{t}{T}} - e^{-\ln 2 \cdot \frac{t}{T'}} \right)$$

$$= 1{,}65 \times 10^{-3}\ g = 1{,}65\ mg$$

c) $\frac{dz}{dt} = \lambda' y = \frac{\lambda' \lambda x_0}{\lambda' - \lambda} \left(e^{-\lambda t} - e^{-\lambda' t} \right)$

$$z(t) = \frac{\lambda' \lambda x_0}{\lambda' - \lambda} \int \left(e^{-\lambda t} - e^{-\lambda' t} \right) dt$$

$$= \frac{\lambda \lambda' x_0}{\lambda' - \lambda} \left(\frac{e^{-\lambda t}}{-\lambda} + \frac{e^{-\lambda' t}}{\lambda'} \right) + K_3$$

$t = 0$, $z = 0$ d'où :

$$K_3 = -\frac{\lambda \lambda' x_0}{\lambda' - \lambda} \left(-\frac{1}{\lambda} + \frac{1}{\lambda'} \right) = x_0$$

$$z(t) = x_0 + \frac{\lambda \lambda' x_0}{\lambda' - \lambda} \left(\frac{e^{-\lambda' t}}{\lambda'} - \frac{e^{-\lambda t}}{\lambda} \right)$$

d) $x + y + z =$

$$x_0 e^{-\lambda t} + \frac{\lambda x_0}{\lambda' - \lambda} \left(e^{-\lambda t} + e^{-\lambda' t} \right)$$

$$+ x_0 + \frac{\lambda \lambda' x_0}{\lambda' - \lambda} \left(\frac{e^{-\lambda' t}}{\lambda'} - \frac{e^{-\lambda t}}{\lambda} \right)$$

$$= x_0 e^{-\lambda t} + \frac{\lambda x_0}{\lambda' - \lambda} e^{-\lambda t} - \frac{\lambda x_0}{\lambda' - \lambda} e^{-\lambda' t}$$

$$+ x_0 + \frac{\lambda x_0 e^{-\lambda' t}}{\lambda' - \lambda} - \frac{\lambda' x_0}{\lambda' - \lambda} e^{-\lambda' t}$$

$$= x_0 e^{-\lambda t} - x_0 e^{-\lambda' t} + x_0 = x_0$$

Exercice 4

$$A(t) = A_0 e^{-\lambda t}$$

$$\frac{A(t)}{A_0} = e^{-\lambda t} \quad \text{et} \quad \lambda t = \ln \frac{A_0}{A}$$

$$t = \frac{1}{\lambda} \ln \frac{A_0}{A} = \frac{t_{1/2}}{\ln 2} \times \ln \frac{A_0}{A}$$

$$= 12279\ a$$

Exercice 9

a) $A_0 = \lambda N_0 = \frac{\ln 2}{t_{1/2}} \times \frac{m_0}{M} N_A$

$$m_0 = \frac{A_0 \cdot t_{1/2} \cdot M}{\ln 2 \times N_A} = \frac{40 \times 10^6 \times 3{,}7 \times 10^{10} \times}{\ln 2 \times 6{,}02 \times 10^{23}}$$

$$\frac{8{,}04 \times 24 \times 3600 \times 131}{1} = 2{,}7 \times 10^{-11}\ g$$

b) $A = \lambda N \times 0{,}35 = 0{,}35 \lambda N_0 e^{-\lambda t}$

$$= 0{,}35 \times A_0 \times e^{-\ln 2 \times t / t_{1/2}} = 10.65\ \mu Ci$$

c) $\frac{dD}{dt} = \frac{1}{m} \frac{dE}{dt} = \frac{E_e}{m} \frac{dN_e}{dt} = \frac{0{,}35\, E_0}{m} \times \lambda N(t)$

$$= \frac{0{,}35\, E_e}{m} \lambda N_0 e^{-\lambda t} = \frac{0{,}35 A_0 E_0}{m} e^{-\lambda t}$$

$$D = \frac{0{,}35 A_0 E_e}{m} \int_0^t e^{-\lambda t}\, dt = \frac{0{,}35 A_0 E_0}{m \lambda} \left(1 - e^{-\lambda t} \right)$$

$$= \frac{0{,}35 \times A_0 \times E_0 \times t_{1/2}}{m \times \ln 2} \left(1 - e^{-\ln 2\, t / t_{1/2}} \right) = 0.18\ Gy$$

$\rightarrow H = 0{,}18\ Sv$

d) $(-dN)_{eff} = (-dN)_r + (-dN)_b$

$$\frac{1}{T_{eff}} = \frac{1}{T_r} + \frac{1}{T_b} \Rightarrow T_b = \frac{T_r T_{eff}}{T_r - T_{eff}}$$

$$= \frac{8{,}04 \times 5}{8{,}04 - 5} = 13{,}2\ j$$